UNIVERSITY OF HULL

Differential Geometry (661955)

Trimester 1

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Motivation

Topology and Differential Geometry

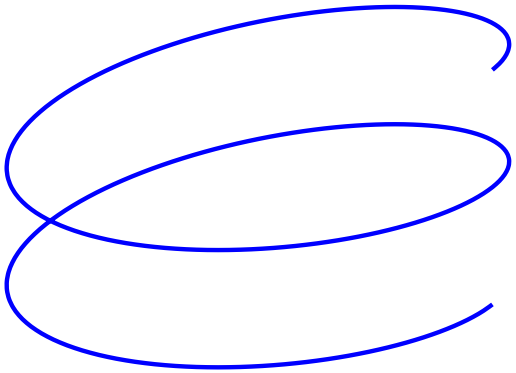
- Both deal with the study of shape:
  - Topology from a continuous POV
  - Differential geometry from a differentiable POV
- Are the languages of lots of modern mathematics and physics
  - Functional Analysis *(notions of weak convergence)*
  - PDEs on manifolds *(surfaces)*
  - Mechanics of solids
  - General relativity

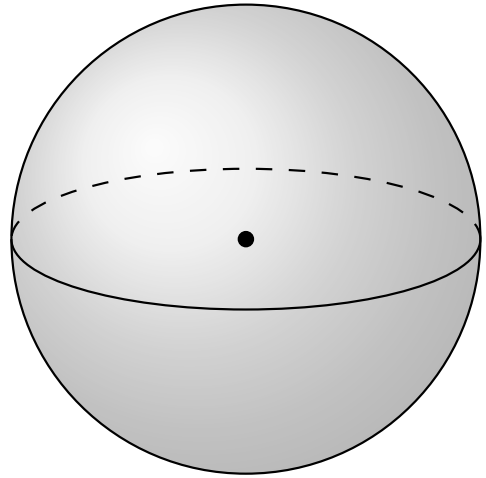
Module content

Divided in 3 parts:

- Curves
- Surfaces
- General topology

Curves & Surfaces





- Curves are 1D and Surfaces 2D *(helix – sphere)*
- Diff Geometry: Study C & S from differentiable POV

  Take some derivatives $\rightsquigarrow$ define some scalars: **Curvatures**

- Curvatures measure how much an object deviates from being **flat**

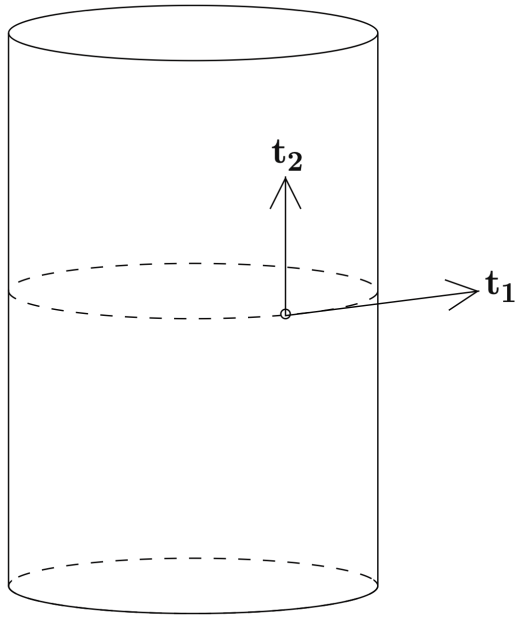
Example: Gaussian Curvature

Principal Curvatures κ_1, κ_2**:** Min and Max curvatures of a surface

Gaussian Curvature: $K = \kappa_1 \kappa_2$

Example: Cylinder of radius R

- Min curvature is $\kappa_1 = 0$; Max curvature is $\kappa_1 = 1/R$
- Gaussian curvature is $K = \kappa_1 \kappa_2 = 0$



One of the best Theorems in Differential Geometry (we will study it)

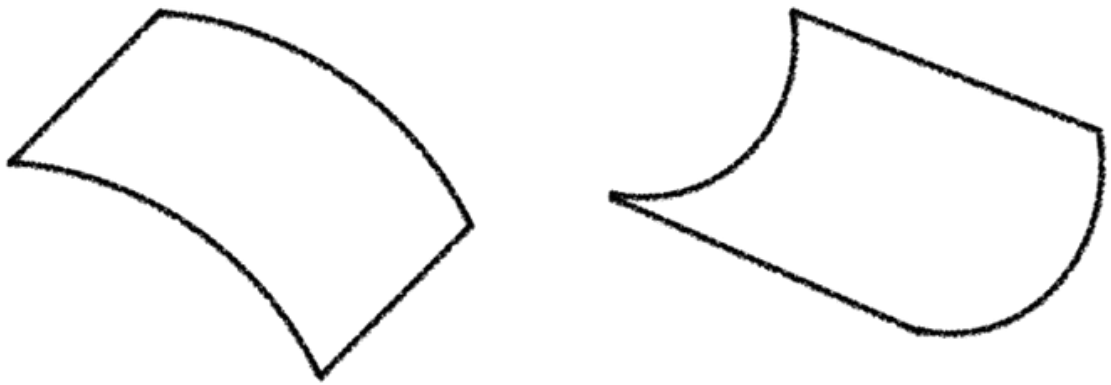
Theorema Egregium: Take a surface

- Bend it in any way
- Don't stretch, shrink or tear it

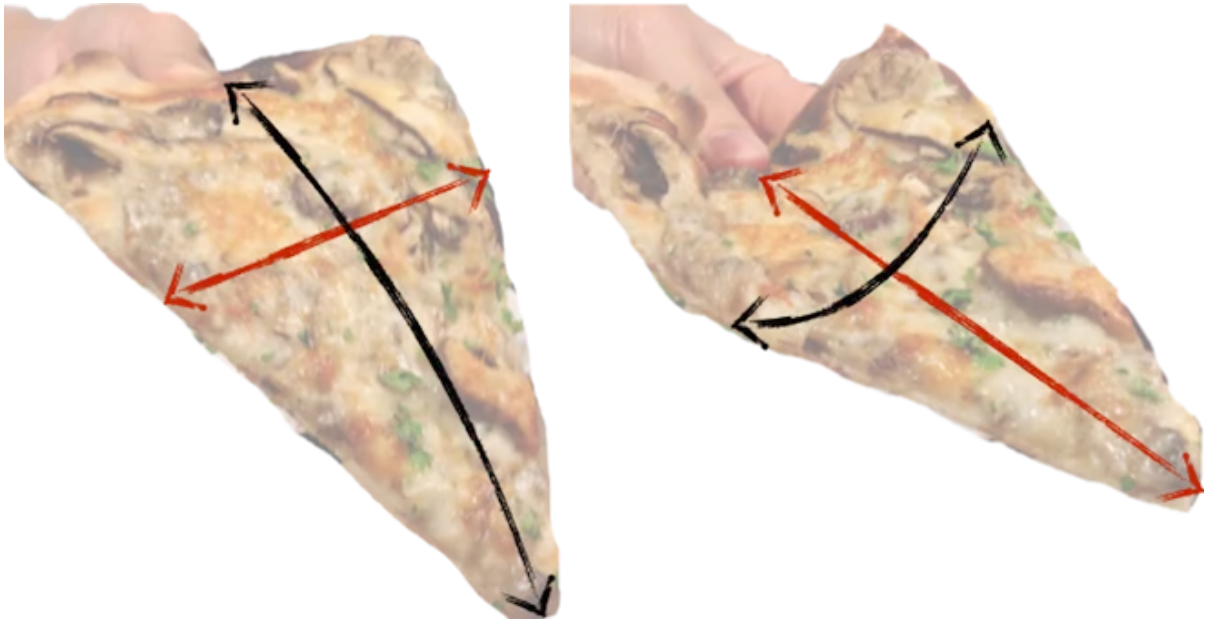
$\Longrightarrow$ Gaussian curvature K stays the same

Example: Piece of Paper is flat $\Longrightarrow$ Gaussian curvature $K = 0$

- Bend it $\Longrightarrow$ 1 direction must stay **flat** to preserve $K = 0$ curvature



Important Application



Goal: Eating pizza

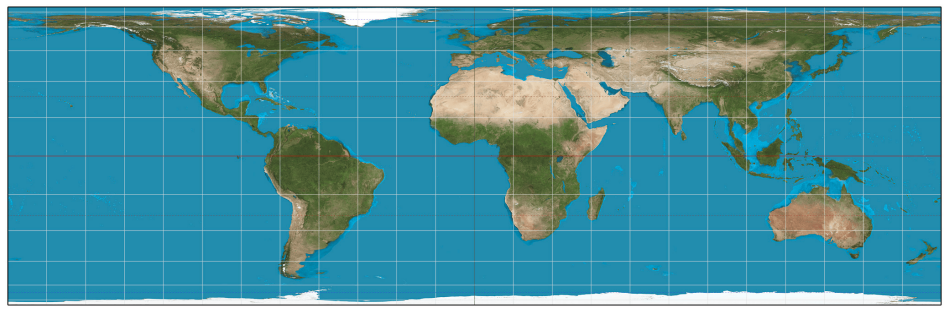
- Slice of pizza on plate is flat $\implies K = 0$
- Lift pizza slice $\implies$ one direction stays flat to preserve $K = 0$
- Bend crust $\implies$ opposite direction flattens to preserve $K = 0$

More serious Application: Cartography

- Model the Earth with a Sphere of Radius R
- Gaussian curvature of Earth is $K = 1/R^2 \neq 0$
- Gaussian curvature of Piece of Paper is $K = 0$

Thm Egregium $\Longrightarrow$ Earth cannot be mapped onto a plane without distorting **Angles** or **Areas**

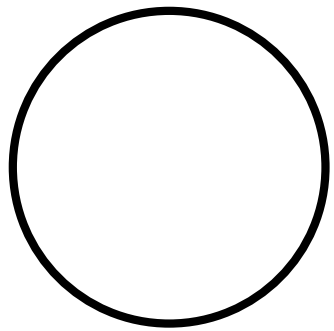
Mercator projection distorts Areas



Lambert projection distorts Angles

Topology

- Study shapes from continuous POV
- 2 shapes are topologically the same:
  - If you can deform one into the other
  - You can bend, stretch, shrink
  - You cannot tear!
- **Question:** Are the shapes below topologically the same?



Module organization

Teaching: Each week we have

- 2 lectures of 2 hours
- 1 tutorial of 1 hour

Assessment:

- 4 problem sheets (accounts for 30% of final mark)
- Exam (accounts for 70% of final mark)

Get in touch for more information:

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